Linda Mei

Final Project: Piece 4 Three Distributions Essay

The normal distribution is the most used continuous probability distribution, and one can tell that it is the normal distribution because it looks like a bell curve. The normal density function has the parameters or symbols mu () and sigma (). Mu indicates the center of the distribution (the mean) while sigma marks the spread of the distribution (the standard deviation). The normal density function has no closed form, it goes from negative infinity to positive infinity. So if one wants to get the area under the normal density function or the bell curve, one will need to integrate the function across an interval from a to b.

Normal distributions can also be used for random variables. A normally distributed random variable with parameters and will use the formulas for the expected () and variance (). One can use a table to find the probabilities and quantiles for normal distributions with random variables. Additionally, one can use software to solve the normal distributions by using commands instead of solving the distribution by hand. The only difference is the software will be more accurate because there are more decimal places, though it does not change much else. A shortcut for the bell curve graph is that it is symmetrical. So one can get the values for both sides of the graph by getting either the left or right half of the bell curve’s values. For example, points -2 and 2 have the same distance from the center of the distribution so they will have the same f(y) value. One can also turn a normal random variable into a standard normal variable by using the formula and plugging in values for , , and are either the a or b of an interval.

Gamma probability distributions are nonsymmetric in which data gets skewed to the right. In the graph, as x increases and gets farther away from the origin, y will drop. The gamma function is denoted by Γ() and it can do factorials for more complex numbers. The two parameters and can change how the gamma density function appears. The parameter is called the shape parameter because it changes the shape of the gamma density. The parameter is called the scale parameter because it is a multiplier that alters the gamma density’s scale without changing its shape.

There is a special case in which a gamma distribution can be expressed as a sum of certain Poisson probabilities where is an integer. If is not an integer and the intervals c and d are not positive, there is no closed form for the gamma distribution. Essentially, the best way to calculate probabilities with the gamma distribution is to use digital software. This is because for values not equal to 1, it is impossible to directly integrate the gamma distribution’s integral. The gamma distribution also has an expected () and variance ().

Another distribution related to the gamma distribution is the chi-square distribution or chi-square () random variable. If the gamma distribution with a random variable has parameters α = and then it is a chi-square distribution with degrees of freedom. The chi-square random variable with v degrees of freedom also has an expected () and variance (). A gamma density function with a parameter equal to 1 is called the exponential density function. The exponential density function can be used to model the length of life (how long it will operate) of electronic components (e.g. fuse). The exponential density function also has an expected () and variance (). A property of the exponential distribution is called the memoryless property (geometric distribution also has this property). An example of the memoryless property can be described as a lock in which the previous attempts of opening the lock do not affect the current attempt of opening a lock, so the probabilities will always be the same on each attempt.

The beta probability distribution has a beta density function with a closed interval from zero to one. The beta probability distribution models proportions such as the impurities in chemical products or a machine’s repair time. The parameters of a beta probability distribution are the positive parameter values and . Depending on the parameter values, the graph can have a different shape. Essentially, one can translate the function over an interval or change the scale. Additionally, the incomplete beta function is the cumulative distribution function for the beta random variable which also has an expected ()and variance (. However, to integrate the beta density function, both and have to be integers.

Density functions model for populations of real data that occur randomly. One does not need to find the “perfect” density function for a model. The density function simply has to yield good probability statements or inferences about the population of interest. The first method is to act on theoretical considerations, meaning that based on the situation one assesses what is known and then decides the best fit. Another way is to form a frequency histogram to select the model (a normal distribution would best model the bell-shaped frequency distribution of the population). One can also use statistics to measure the goodness of fit among all distributions to find the best fit.

For density functions, there are also other expected values or moments for continuous random variables. There is a moment about the origin and a central moment. There is also a moment-generating function of a random variable . Moments can describe the data through numerical descriptive measures in an experiment. One can also use moments to prove that a random variable has a specific probability distribution like the gamma-distributed random variable in example 4.13. If two (discrete or continuous) random variables have the same moment-generating functions they have the same probability distributions.

Tchebysheff’s theorem was mentioned in 3.11 it talks about finding the lower bound of the probability that the random variable falls in standard deviations of the mean. One can also apply this theorem to continuous random variables. Tchebysheff’s theorem makes it easier to find the bounds for probabilities that would normally require integration or summation. One can get the mean and variance of random variables without the distribution of the variable. With Tchebysheff’s theorem, one can skip evaluating an integral. They can plug in the values for the theorem and get the probability of interest. Afterward, they can consider what the probability means to form a conclusion. Some of these integrals cannot be evaluated directly by hand and require one to use tables or software. Essentially, one can use Tchebysheff’s theorem to find quick bounds of probabilities or the mean and variance of random variables.